Second Order Devices

**Capacitor**

A device which stores energy in an electric field. Comprised of two conductive "plates" separated by a dielectric medium. Charge stored on the plates is equal to the product of the voltage across the plates and the capacitance of the device.

\[ q = CV_c \rightarrow \frac{dq}{dt} = \frac{1}{C} (CV_c) \rightarrow \frac{dv_c}{dt} = C \frac{dv_c}{dt} \]

\[ \frac{dv_c}{dt} = C^{-1} i_c \rightarrow \int \frac{dv_c}{dt} dt = \int C^{-1} \frac{dv_c}{dt} dt \rightarrow v_c = C^{-1} \int i_c dt \]

**Reactance is frequency dependent and of the form**

\[ X_c = \frac{1}{2\pi f C} = \frac{1}{\omega C} \]

We can also investigate the complex impedance in the Laplace domain by starting with one of our \( v_c \) or \( i_c \) equations in the time domain:

\[ v_c(t) = \frac{1}{C} \int i_c dt \]

\[ \mathcal{L}\{v_c(t)\} = \mathcal{L}\left\{ \frac{1}{C} \int i_c dt \right\} \]

\[ \mathcal{L}\{v_c(s)\} = \frac{1}{C} \mathcal{L}\{\int i_c dt\} \]

\[ V_c(s) = \frac{1}{C} \mathcal{L}\{i_c\} \]

\[ V_c(s) = \frac{1}{C} \times \frac{1}{s} \times I_c(s) \]

\[ \frac{V_c(s)}{I_c(s)} = \frac{1}{sC} \]

\[ Z_c(s) = \frac{1}{sC} \]