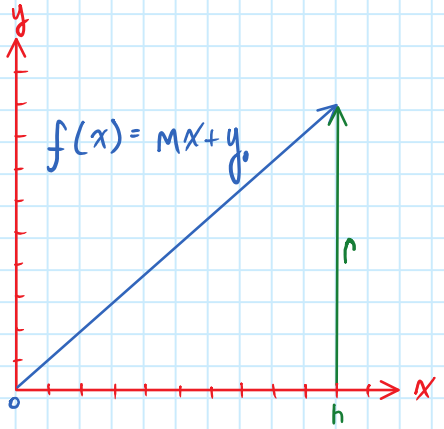
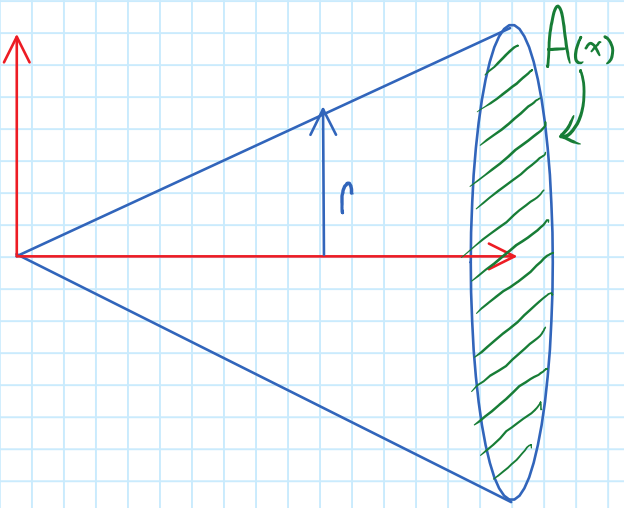
Volume of Cone Via Solid of Revolution



Revolve About x-axis



$$A(x) = \pi r^2(x) = \pi f^2(x)$$

(or simplifying using similar triangles)

$$A(x) = \pi\left(\frac{x}{h} \cdot r\right)^2 = \pi r^2 \frac{x^2}{h^2}$$

$$V = \int_0^h A(x)\,dx$$

$$= \int_0^h \pi r^2 \frac{x^2}{h^2}\,dx$$

$$= \frac{\pi r^2}{h^2} \int_0^h x^2\,dx$$

$$= \frac{\pi r^2}{h^2}\left[\frac{1}{3}x^3\right]_0^h$$

$$= \frac{\pi r^2}{h^2}\left[\frac{1}{3}h^3 - 0\right] = \frac{1}{3}\pi r^2 h$$