Second Order Devices

Capacitor

A device which stores energy in an electric field. Comprised of two conductive "plates" separated by a dielectric medium. Charge stored on the plates is equal to the product of the voltage across the plates and the capacitance of the device.

$$q = Cv_c \rightarrow \frac{dq}{dt} = \frac{d}{dt}(Cv_c) \rightarrow \boxed{i_c = C\frac{dv_c}{dt}}$$

$$\frac{dv_c}{dt} = C^{-1}i_c \rightarrow \int \frac{dv_c}{dt}\,dt = \int C^{-1}i_c\,dt \rightarrow \boxed{v_c = C^{-1}\int i_c\,dt}$$

Reactance is frequency dependant and of the form

$$X_c = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

We can also investigate the complex impedance in the Laplace domain by starting with one of our v_c or i_c equations in the time domain:

$$v_c(t) = \frac{1}{C}\int i_c\,dt$$

$$\mathcal{L}\{v_c(t)\} = \mathcal{L}\left\{\frac{1}{C}\int i_c\,dt\right\}$$

$$V_c(s) = \frac{1}{C}\mathcal{L}\left\{\int i_c\,dt\right\}$$

$$V_c(s) = \frac{1}{C}\times\frac{1}{s}\times I_c(s)$$

$$\frac{V_c(s)}{I_c(s)} = \frac{1}{sC}$$

$$\boxed{Z_c(s) = \frac{1}{sC}}$$