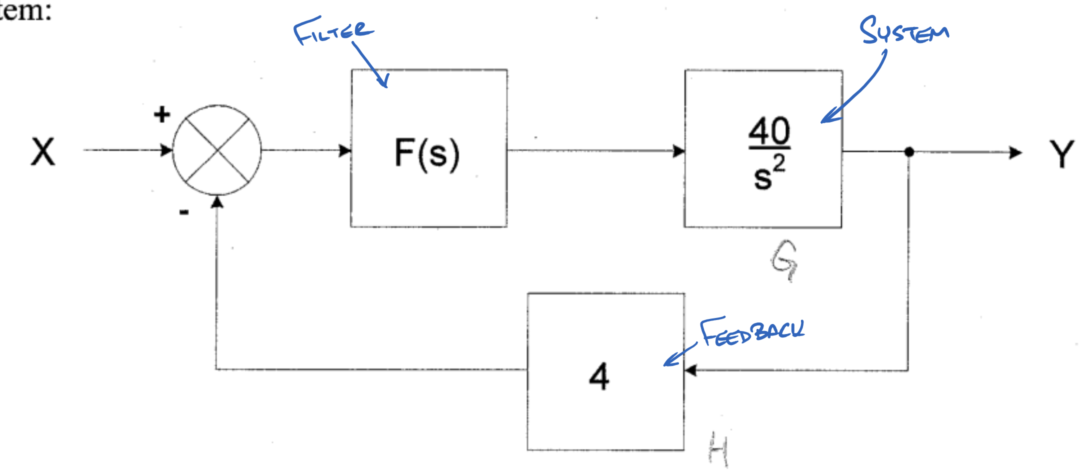
Phase Margin Compensation Using a Lead-Lag Filter

Design a lead-lag filter which will give a phase margin of **60 degrees** when placed in the box labeled F(s) in the following system:



① Transfer Function of System

$$GH(s) = \frac{40}{s^2} \times 4 = \frac{160}{s^2}$$

$$|GH(j\omega)| = \left|\frac{160}{(j\omega)^2}\right| = \left|-\frac{160}{\omega^2}\right| = \frac{160}{\omega^2} = 1 \Big|_{\omega=\omega_c} \Rightarrow \omega_c = \sqrt{160} = \boxed{12.6\ \text{RAD/s}}$$

② Transfer Function of Lead-Lag Filter

$$F(s) = \frac{1}{\sqrt{M}} \times \frac{1 + s/\omega_z}{1 + s/\omega_p}$$

$$M = \left[\tan\phi_{MAX} + \sqrt{\tan^2\phi_{MAX} + 1}\right]^2$$

$$= \left[\tan 60^\circ + \sqrt{\tan^2 60^\circ + 1}\right]^2 = \boxed{13.9}$$

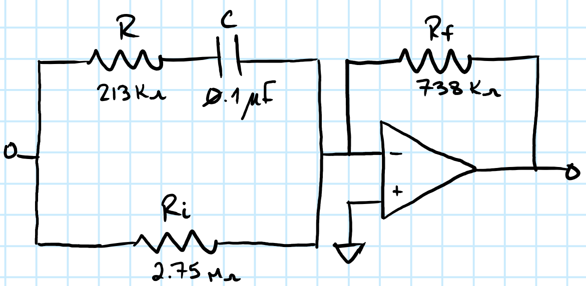
$$\omega_c = \omega_{MAX} = 12.6\ \text{RAD/s}$$

$$\omega_z = \frac{\omega_{MAX}}{\sqrt{M}} = \boxed{3.38\ \text{RAD/s}}$$

$$\omega_p = \omega_{MAX}\sqrt{M} = \boxed{47.0\ \text{RAD/s}}$$

$$\boxed{F(s) = 0.268\,\frac{1 + s/3.38\frac{\text{RAD}}{s}}{1 + s/47.0\frac{\text{RAD}}{s}}}$$

③ Design an Analog Control Circuit for This Transfer Function (Inverting)



Assume $C = 0.1\,\mu F$

$$R = \frac{1}{\omega_p C} = \frac{1}{(47\frac{\text{RAD}}{s})(0.1\,\mu F)} = \boxed{213\ K\Omega}$$

$$R_f = \frac{R_i}{\sqrt{M}} = \frac{2.75\,M\Omega}{3.9} = \boxed{738\ K\Omega}$$

$$R_i = \frac{1}{\omega_z C} - R = 2.77\,M\Omega - 213\,K\Omega = \boxed{2.75\,M\Omega}$$